# The space of metric structures on hyperbolic groups

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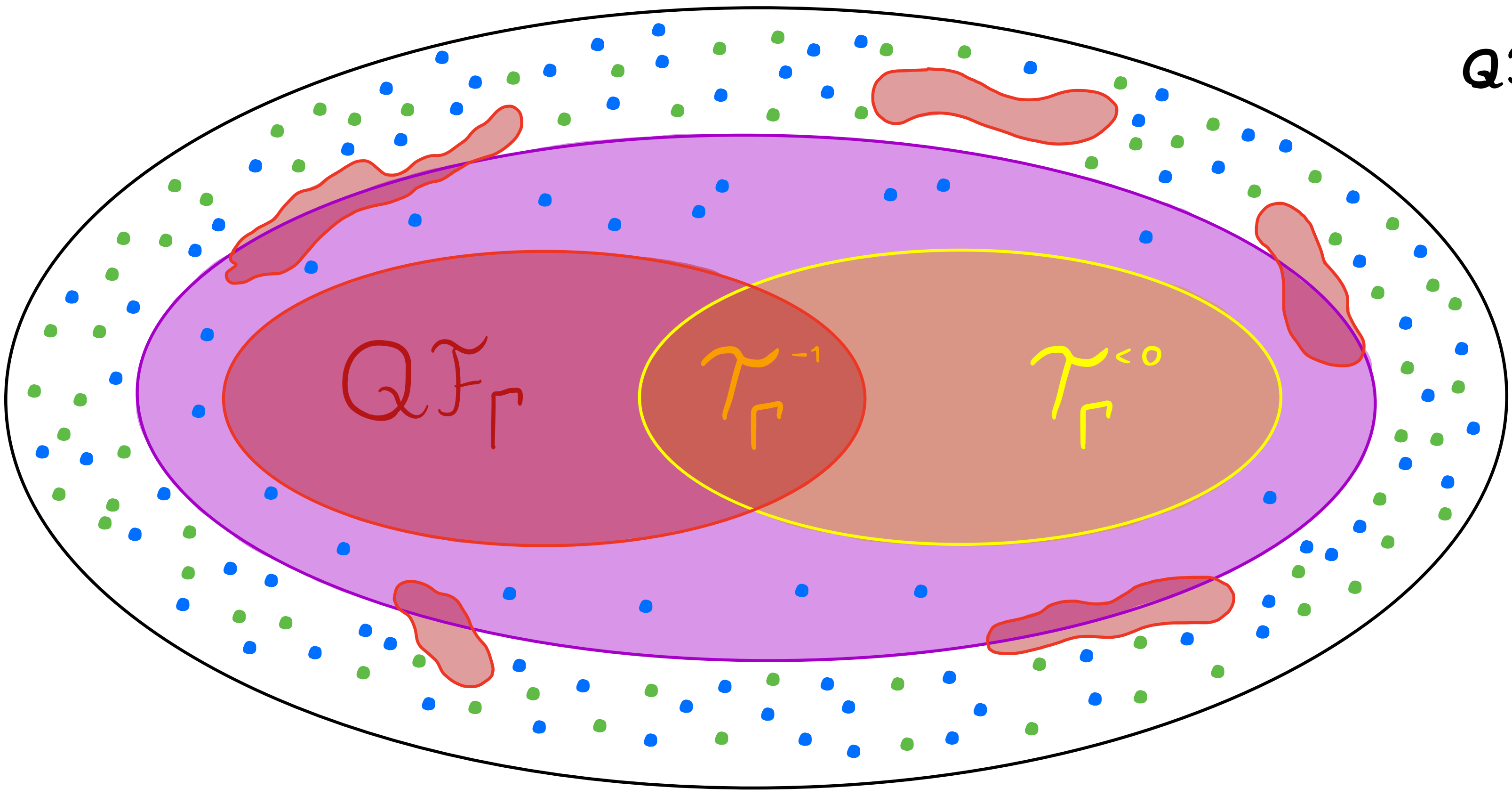
**Motivation:** Many interesting deformation spaces for group actions

e.g. Teichmüller space, Outer space, Character varieties, spaces of Riemannian metrics

$\Gamma$ non-elementary hyperbolic group $\rightsquigarrow$ $\mathcal{D}_\Gamma$ space of metric structures : parametrizes geometric actions of $\Gamma$ on Gromov hyperbolic spaces (it is a metric space!!!)

**Example:** $\mathcal{D}_\Gamma$ for $\Gamma$ a surface group




$\mathcal{QF}_\Gamma$: Quasi-Fuchsian reps

$\mathcal{T}_\Gamma^{-1}$: Teichmüller space

$\mathcal{T}_\Gamma^{<0}$: Neg. curved metrics

Green metrics

Word metrics

$\overline{\text{cubulations}}$

Anosov reps

**Properties of $\mathcal{D}_\Gamma$** (Cantrell & R., '22+'23)

- Contractible, separable, not locally compact
- Unbounded, geodesic, has a nice boundary
- Topologically homogeneous, all spheres are homeomorphic

## Applications (Cantrell + R., '22 + '23)

1] **Counterexamples to a conjecture of Bonahon '88:** $\exists\ \Gamma \curvearrowright T$, isometric, $T$ tree st:

i) Action is not small, and

ii) Translation length extends continuously to currents

2] **Gap for marked length spectrum rigidity:** New proofs for

**Coro 1** (Gogolev - Rodriguez Hertz): $\Gamma = \pi_1(S)$, $S$ closed surface $g, g_*$ neg. curved metrics

$\ell_g(\gamma) = \ell_{g_*}(\gamma)$ $\forall \gamma$ geodesic in fixed homology class $\Rightarrow$ $g, g_*$ isometric

**Coro 2** (Dahmani - Futer - Wise): $\Gamma \curvearrowright X$ geometric, $o \in X$, $H < \Gamma$ quasiconvex $|\Gamma : H| = \infty$

$$\lim_{n\to\infty} \frac{1}{n} \log \#\{g \in H / d_X(go,o) \leq n\} < \lim_{n\to\infty} \frac{1}{n} \log \#\{g \in \Gamma / d_X(go,o) \leq n\}$$